

Modeling and optimal design of Planar Linkage Mechanism of Coupled Joint Clearances for Manufacturing

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Abstract: The uncertainty of the mechanism motion error is mostly caused by the manufacturing process, the motion error cannot be effectively predicted at the design phase. The problems of manufacturing complexity and the relationship between design and manufacturing are analyzed, the influence of dimensional tolerance and fit tolerance on the motion accuracy of the system is considered in the design process, then based on the Monte Carlo simulation, the planar linkage mechanism optimal design model is set up. A typical offset slider-crank mechanism is used as an illustrative example to carry out the optimal design. Compared with the result of the typical robustness design, the similar variation characteristics of the mean value and the standard deviation can be found, so the method in this paper is effective. The method is furthermore applied in the optimization of the schemes with different fit tolerances, the prediction of motion errors in the design phase is achieved. A set of the quantitative evaluation system for mechanism optimal design is provided. At the end of this paper, a basic thought of balancing the motion precision and the cost of manufacturing is presented.

Key words: Manufacturing; Planar linkage mechanism; Monte Carlo Simulation; Dimensional tolerance; Fit tolerance; Balance

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0 Introduction

Planar linkage mechanisms are widely used in machines to ensure their functions. Some motion errors in these machines are inevitable due to tolerance, defects arising from the design and manufacturing process or wearing after a certain working period. Thus, because of these uncertainty factors, the motion error of the mechanism cannot be effectively predicted in the design phase. In the past, the research on the motion error prediction in the presence of uncertainty factors generally can be divided into two kinds of methods.

The first is research on link dimensional errors. Link dimensional error is considered in the design phase, the motion error of the actual mechanism can be reduced. Dong Yuge et al. [1-2] have presented fuzzy reliability analysis of slider-crank mechanism, a method of fuzzy reliability design

was got. However, it is not enough just to precisely predict the motion error of the post-manufacturing's mechanism.

Moreover, joint clearance to take into consideration. S.J.LEE and B.J.Gilmore [3] have presented the joint clearance bar model, and the joint clearance was treated as a massless virtual link with the length equal to one-half of the clearance. Song Li et al. [4] have presented the calculation method of the joint clearance bar dimension distribution. Tan Xiaolan et al. [5] have presented an effective model to analyze the offset slider-crank mechanism with joint clearance, then based on robustness design, the optimal design method was got. Kang Luo and Xiaoping Du [6-9] have presented the modified First Order Second Moment based on the First Order Second Moment and the Monte Carlo simulation method, a four-bar mechanism, and a slider-crank mechanism were used to test the proposed method. Gunesh R et al. [10] aimed at the

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four-bar mechanism and the three different objective functions were got. The reliable foundation for the establishment of the model was provided. Jianmin Zhu and Kwun-Lon^[11-12] have presented a general approach for the kinematics of four-bar mechanism with joint clearances, the assumption of continuous contact model was used. However, a single DOF four-bar linkage would become a five DOF eight-bar linkage and it will inevitably cause the whole model solves tedious. Thus, Guo Huixin et al.^[13-14] have presented the "effective length model" to replace the "joint clearance bar model", and the precision of the model was verified by an offset slider-crank mechanism example.

In this paper, based on the Monte Carlo simulation method, according to the requirement of motion precision, fit between hole and shaft is considered, the distribution range of any joint clearances bar size can be calculated. Combined with the dimensional tolerance of the component for parameter optimization, compared with the mean and standard deviation of the motion error, a set of the quantitative evaluation system for mechanism optimization design is provided. At the end of this paper, a basic thought of balancing the motion precision and the cost of manufacturing is presented.

1 Multi-objective optimal design model of planar linkage mechanism for manufacturing

Actual motion error of the mechanism can be described as Eq. (1).

$$\Delta = y_{am} - y_m \quad (1)$$

Where y_m and y_{am} denote the required motion output and actual motion output, respectively.

Currently, practical mechanism most adopted a serial mode of design to manufacturing, the complexity of the manufacturing process and the relationship between design and manufacturing lead to the practical mechanism inevitable existence some uncertain factors, the uncertainty of the whole motion error increased^[6-12], thus, the motion error of the actual mechanism is

$$\begin{aligned} \Delta(\mathbf{X}, \mathbf{Z}) &= y'_{am}(\mathbf{X}, \mathbf{Z}) - y_m, \\ \mathbf{X} &= x_1, x_2, \dots, x_p, \mathbf{Z} = z_1, z_2, \dots, z_q \end{aligned} \quad (2)$$

Where \mathbf{X} denotes a collection of control factors for the mechanism, \mathbf{Z} denotes a collection of noise factors for the mechanism, p is the number of control factors,

q is the number of noise factors. $y'_{am}(\mathbf{X}, \mathbf{Z})$ is the actual motion of the mechanism, which noise factors and other objective factors are considered. Noise factors includes environmental factors change, manufacturing parameter change, material aging, abrasion, processing method change etc.

Optimization of the usual mechanism is trajectory optimization, namely, there are many discrete points in the trajectory and existence multiple point motion errors, so the mechanism's motion error δ expressed as

$$\delta = \sqrt{\frac{1}{s} \cdot \sum_{n=1}^s (y'_{am}(\mathbf{X}, \mathbf{Z}) - y_m)^2} \quad (3)$$

Where s is the number of discrete points.

Therefore, the objective function of the optimal design model of the planar linkage mechanism for the manufacturing is

$$\min F(\mathbf{X}, \mathbf{Z}) = \omega_1 \mu_\delta + \omega_2 \sigma_\delta \quad (4)$$

Where the mean value of the error δ is μ_δ , and the standard deviation is σ_δ . ω_1 , ω_2 are the weight coefficients for mean and standard deviation^[15].

According to the different application conditions and role of the different caused the diversity of forms, therefore, there is a certain relationship between the link of mechanism f_i (such as rotation characteristics of the whole cycle, the crank existence conditions and the characteristics of minimum transmission angle), f_{ii} are the requirements for the characteristics of the mechanism in Eq.(5).

$$\begin{aligned} f_i(\mathbf{X}, \mathbf{Z}) - f_{ii} &\leq 0, \\ \mathbf{X} &= x_1, x_2, \dots, x_p, \mathbf{Z} = z_1, z_2, \dots, z_q, \\ i &= 1, 2, \dots, n \end{aligned} \quad (5)$$

Where n is the number of requirements for the mechanism's characteristics.

For other objective factors, (include precision, cost, stability, dynamic mechanical behavior etc.) which can furthermore add to a number of the corresponding constraints, to filter the data from design result, specified in Eq. (6).

$$\begin{aligned} g_j(\mathbf{X}, \mathbf{Z}) - g_{ij} &\leq 0, \\ \mathbf{X} &= x_1, x_2, \dots, x_p, \mathbf{Z} = z_1, z_2, \dots, z_q, \end{aligned}$$

$$j = 1, 2, \dots, m \quad (6)$$

Where g_{ij} are the constraints value on other objective factors (determined by the objective requirements), m is the number of other objective factor constraints.

Finally, the optimal design model of the planar linkage mechanism for manufacturing can be expressed as follows.

$$\begin{aligned} \min F(\mathbf{X}, \mathbf{Z}) &= \omega_1 \mu_\delta + \omega_2 \sigma_\delta \\ \text{St. } f_i(\mathbf{X}, \mathbf{Z}) - f_{ii} &\leq 0, i = 1, 2, \dots, n \\ g_j(\mathbf{X}, \mathbf{Z}) - g_{ij} &\leq 0, j = 1, 2, \dots, m \\ \mathbf{X} &= x_1, x_2, \dots, x_p, \mathbf{X} \in (\mathbf{X}^L, \mathbf{X}^U)^T \\ \mathbf{Z} &= z_1, z_2, \dots, z_q, \mathbf{Z} \in (\mathbf{Z}^L, \mathbf{Z}^U)^T \end{aligned}$$

Where \mathbf{X}^L , \mathbf{X}^U are collections of control factors' lower bound and upper bound of value, respectively (determined by initial conditions). \mathbf{Z}^L , \mathbf{Z}^U are collections of noise factors' lower bound and upper bound of value, respectively (jointly determined by design and processing requirements).

2 Modeling of planar four-bar linkage with joint clearance and case analysis

For planar four-bar mechanism, it is well known that the performance measure for a mechanism is usually referred to as the ability to reaching the desired position or orientation precisely. Link dimensional tolerances and also fit tolerances make the performance of mechanisms worse [11-14]. The collection of control factors includes length and it's range, rotation angle and so on. The collection of noise factors includes fit tolerance and dimensional tolerance, geometric tolerance, roughness, the deviation between holes etc. Due to the randomness of the tolerance distribution, which led to the randomness of the motion error. Research on the correlation between tolerance distribution and the model pose, based on the Monte Carlo simulation method, then the mathematical model of optimal design is set up. The offset slider-crank mechanism is used as an illustrative example.

2.1 Pose description of the model of planar four-bar linkage with joint clearance

The four-bar mechanism is widely used in planar linkage mechanism [16]. According to LEE S J's "joint clearance bar model" [3], the hole-shaft structure is assumed for each link, namely, the point A of the frame is the hole,

the point A of the crank is the shaft, the point B of the crank is the hole, the point B of the link is the shaft. The specific pose description of mechanism with joint clearance bar as shown in Fig.1.

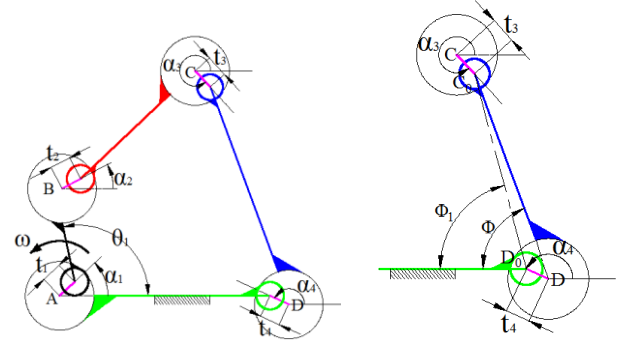


Fig. 1 Pose description with joint clearance bar

Replacing the revolute pair with joint clearance bar that forms a model of an eight-bar linkage mechanism, causing the whole model solves complex. According to Refs [13], the effective length model was adopted (shown in Fig.2, the link CD in Fig.1 is used to describe), replacing $\overrightarrow{C_0D_0}$ with $\overrightarrow{C_0D} + \overrightarrow{DD_0}$, that is, replacing the effective length with the joint clearance bar plus the original bar. Finally, the four-bar model with the effective length is generated.

The relationship between $\overrightarrow{C_0D}$ and $\overrightarrow{C_0D_0}$ is analyzed, the vector equation can be expressed as Eq.(7).

$$C_0D_0e^{i\Phi_1} = C_0De^{i\Phi} + t_4e^{-i\alpha_4} \quad (7)$$

Where the length of the original bar is C_0D , t_4 is the length of joint clearance bar of revolute pair D, α_4 is the azimuth of joint clearance bar, and C_0D_0 is the length of the effective bar.

The geometric relation can be expressed as Eq. (8).

$$C_0D_0 \begin{bmatrix} \sin \Phi_1 \\ \cos \Phi_1 \end{bmatrix} = C_0D \begin{bmatrix} \sin \Phi \\ \cos \Phi \end{bmatrix} + t_4 \begin{bmatrix} -\sin \alpha_4 \\ \cos \alpha_4 \end{bmatrix} \quad (8)$$

Because $t_4 \ll C_0D$, namely, $\Phi = \Phi_1$.

Take the sum of squares of Eq. (8), then, the Eq. (8) can be expressed as Eq. (9).

$$C_0D_0 = C_0D \sqrt{1 + 2 \frac{t_4}{C_0D} \cos(\Phi + \alpha_4)} \quad (9)$$

Because $t_4 \ll C_0 D \Rightarrow t_4 / C_0 D \rightarrow 0$, according to

$\lim_{x \rightarrow 0} \sqrt[n]{1+x} = 1 + x/n$, then,

$$C_0 D_0 = C_0 D + t_4 \cos(\Phi + \alpha_4) \quad (10)$$

Namely, by **Eq. (10)**, the replacement of the eight bar mechanism to four bar mechanism is realized, and a mathematical model of planar four-bar mechanism with joint clearance is established.

2.2 Analyzing multi-objective optimal design model and solving based on the Monte Carlo simulation method

The components mass production, all the tolerances of the link dimension follow a normal distribution, while the azimuth of joint clearance bar follows a uniform distribution [11-12], predicting the probability distribution model of the whole motion error is difficult. The Monte Carlo simulation method is applied to probability analysis of the multivariate random function, and it commonly is used in tolerance analysis [17-18], in this paper, using this method, the optimal design of planar four-bar mechanism with joint clearance for manufacturing is presented.

2.2.1 Simulating the length of joint clearance bar by the Monte Carlo simulation method

According to the design requirements, after manufacturing, the joint clearance bar naturally forms. The dimension of length t_k (include t_1 , t_2 , t_3 and t_4) can be expressed as **Eq. (11)**.

$$t_k = \frac{1}{2}(d_{hk} - d_{sk}) \quad (11)$$

Supposing the inner diameter of the hole expressed as d_{hkEI}^{ES} , the outer diameter of the shaft expressed as d_{skEI}^{es} , according to the principle of ± 3 standard deviation [11-12], the standard deviation of hole and shaft dimension can be obtained, namely,

$$d_{hk} \sim N(\bar{d}_{hk}, \sigma_{hk})$$

$$d_{sk} \sim N(\bar{d}_{sk}, \sigma_{sk}).$$

The dimension error of the manufacturing follows the normal distribution, according to normal distribution sampling formula,

$$d_{vk} = \bar{d}_{vk} + \sigma_{vk} \sqrt{-2 \ln a} \begin{bmatrix} \cos 2\pi b \\ \sin 2\pi b \end{bmatrix} \quad (12)$$

Where \bar{d}_{vk} is the mean value of the hole or shaft, σ_{vk} is the standard deviation, $v = h, s$ express the hole or shaft respectively, and the parameter $a, b \sim U(0,1)$.

Based on the Monte Carlo simulation method, sampling according to **Eq. (12)**, the sample value of the hole d_{hk} and the shaft d_{sk} are represented as row vectors $d_{hk1}, d_{hk2}, d_{hk3}, \dots, d_{hkN}$ and $d_{sk1}, d_{sk2}, d_{sk3}, \dots, d_{skN}$ (N denotes sample size). According to mathematical statistics method, due to **Eq. (11)**, do some further processing to the obtained samples, the mean value \bar{t}_k and standard deviation σ_{tk} of the length of joint clearance bar t_k can be obtained, that is, $t_k \sim N(\bar{t}_k, \sigma_{tk}^2)$.

2.2.2 Simulating the mechanism's motion error by the Monte Carlo simulation method

Based on the Monte Carlo simulation method, according to the range of control factors and the distribution of noise factors, the sample can be extracted and calculated [17-19] (sample size $N = 10^6$). The description of the mean value and standard deviation of motion error in **Eq. (3)** can be got, combined with constraints of the mechanism's characteristics, through the optimization algorithm (The penalty function method is used in this paper) to solve, and an optimum solution can be got. The optimal design process of planar four-bar mechanism with joint clearance is shown in **Fig.3**.

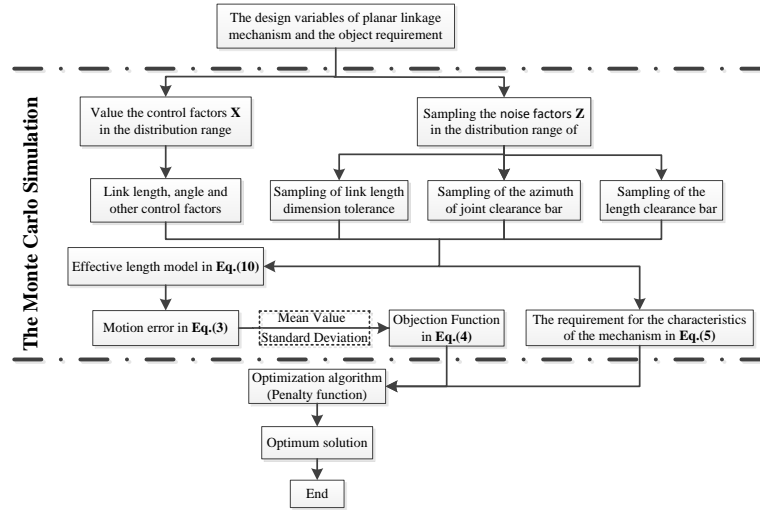


Fig. 3 Planar four-bar mechanism with joint clearance optimal design flow chart

2.3 Optimal design example analyses of offset slider-crank mechanism with joint clearance

Offset slider-crank mechanism as shown in Fig.4, in order to convenient compare with the Refs. [13], the working range of crank is $[225^\circ, 285^\circ]$, and the corresponding working range of slider is $[80, 120]$ mm, the relation between the displacement of the slider s_0 and the angle of the crank θ described in Eq. (13), that is, the required motion output y_{rm} in Eq. (3).

$$s_0 = 80 + 40 \times \left(\frac{\theta - 225^\circ}{60^\circ} \right)^2 \quad (13)$$

Actual motion displacement of the slider calculated by Eq. (14) (denote actual motion output y'_{am} in Eq. (3))

$$s = l_1 \cos \theta + \sqrt{l_2^2 - (l_3 - l_1 \sin \theta)^2} \quad (14)$$

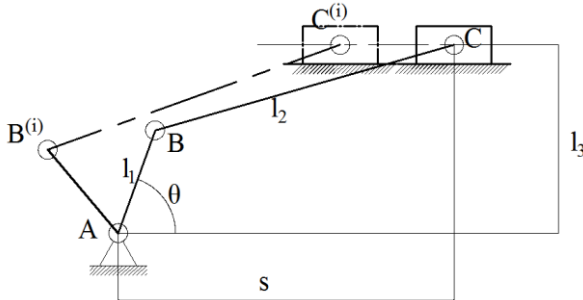


Fig. 4 Offset slider-crank mechanism

13 points θ_i in the angle range $[225^\circ, 285^\circ]$ of the

crank can be got, for $i=1,2,\dots,13$. Therefore, the link dimensional tolerance and the revolute pair fit tolerance were considered at the same time, the motion displacement error of the slider can be expressed as Eq. (15).

$$\delta = \sqrt{\frac{1}{13} \cdot \sum_{i=1}^{13} (s_0(\theta_i) - s(l_1, l_2, l_3, \theta_i, \Delta l_1, \Delta l_2, \Delta l_3, t_1, t_2, t_3, \alpha_1, \alpha_2, \alpha_3))^2} \quad (15)$$

Where $\mathbf{X} = (l_1, l_2, l_3)^T$ are control factors, $\mathbf{Z} = (\Delta l_1, \Delta l_2, \Delta l_3, t_1, t_2, t_3, \alpha_1, \alpha_2, \alpha_3)^T$ are noise factors. $\Delta l_1, \Delta l_2, \Delta l_3$ are dimensional tolerances of crank, link, and offset distance, respectively.

2.3.1 Model establishment of offset slider-crank mechanism

In Refs. [16], supposing the basic dimension of the inner diameter of the hole and the outer diameter of the shaft range from $(10, 18)$ mm, according to the characteristics of fit and conditions of application, from small to large, choosing three fit clearances $(\frac{H8}{g7}, \frac{H7}{f6})$

and $\frac{H8}{e8}$, respectively). To make the following analysis convenient, using 1, 2 and 3 to express the different fit clearance, such as point A is $\frac{H8}{g7}$, point B is $\frac{H8}{e8}$, point C

is $\frac{H7}{f6}$, that is, each joint point of ABC can be described as scheme 132. According to Refs. [16], the value of the limit

deviation of hole and shaft can be got, then, through the method in section 3.2.1, the mean value and the standard deviation of the joint clearance can be obtained, the results are given in Table 1.

Table 1 Mean Value \bar{t}_k and standard deviation σ_{tk} of three different kinds of joint clearance / (mm)

Number	Fit type	\bar{t}_k	σ_{tk}
1	$\frac{H8}{g7}$	0.008 98	0.003 26
2	$\frac{H7}{f6}$	0.018 39	0.002 58
3	$\frac{H8}{e8}$	0.023 18	0.002 76

Control factors and noise factors are given in Table 2, through the principle of 3σ [11-12], the standard deviation of the crank, link, and offset distance can be obtained.

Table 2 The distributed parameter of control factors and noise factors

Design variable ($k=1,2,3$)	Mean value	Standard deviation	Distribution type
l_k / mm	\bar{l}_k	$\Delta l_k / 3$	Normal distribution
t_k / mm	\bar{t}_k	σ_{tk}	Normal distribution
$\alpha_k / ^\circ$	$\bar{\alpha}_k$	$\sigma_{\alpha k}$	$[0, 2\pi]$ Uniform distribution

The optimal design model of slider-crank mechanism with joint clearance can be expressed as follows.

$$\min F(\mathbf{X}, \mathbf{Z}) = \omega_1 \mu_\delta + \omega_2 \sigma_\delta \quad (3)$$

$$\text{St. } l_1 + t_1 - l_2 - t_2 + l_3 + t_3 \leq 0 \quad (16)$$

$$\mathbf{X}^L = (50, 160, 65)^T, \mathbf{X}^U = (60, 170, 75)^T$$

$$\mathbf{Z}^L = (0.001, 0.001, 0.001)^T$$

$$\mathbf{Z}^U = (0.15, 0.15, 0.15)^T$$

Initial Value

$$(55, 165, 70, 0.07, 0.07, 0.07)^T$$

Where we get $\omega_1=1$ 、 $\omega_2=1$ [15]. From line position tolerance [16], the range of dimensional tolerances of \mathbf{Z}^L , \mathbf{Z}^U can be confirmed.

2.3.2 Solution and result analysis of mathematical model

Based on the typical robustness optimal design, aiming at offset slider-crank mechanism, in Refs. [13], a optimal design model with joint clearance was established, and the link dimensional tolerance of the mechanism were got. Aiming at the scheme, in order to contrast the effect of the design, hence, the same dimensional tolerance and fit tolerance are adopted, the design parameter of Refs. [13] and this paper are given in Table 3.

Table 3 The results of this paper compared with robustness design

Design Variables	l_1 / mm	l_2 / mm	l_3 / mm	$\Delta l_1 / mm$	$\Delta l_2 / mm$	$\Delta l_3 / mm$
Original scheme [13]	58.130	165.480	71.790	0.081	0.072	0.093
This paper scheme	58.011	164.787	71.014	0.081	0.072	0.093

The mean value and standard deviation of the error are the important parameters, from which the motion error of the mechanism can be evaluated [6-9, 13-14]. The optimization result is calculated through the design process in Fig.5, and the mean value and standard deviation of motion error can be obtained, respectively (as shown in Fig.6 and Fig.7).

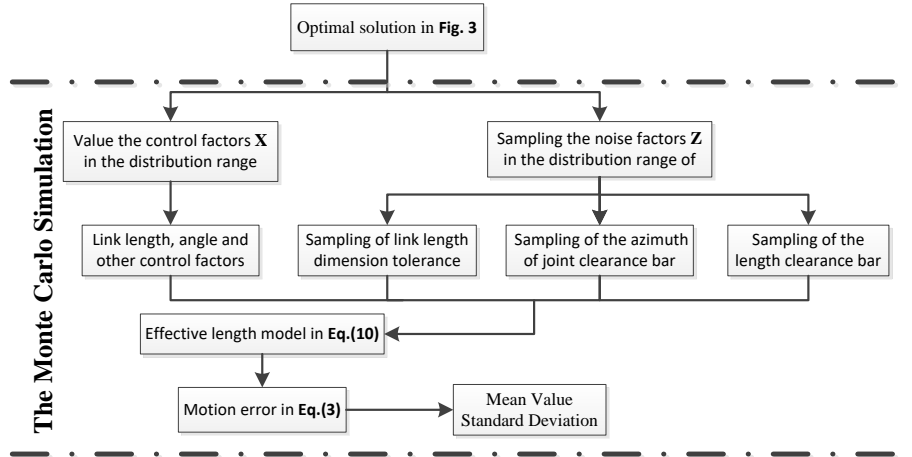


Fig. 5 Calculation flow chart of the mean value and standard deviation of the motion error with joint clearance

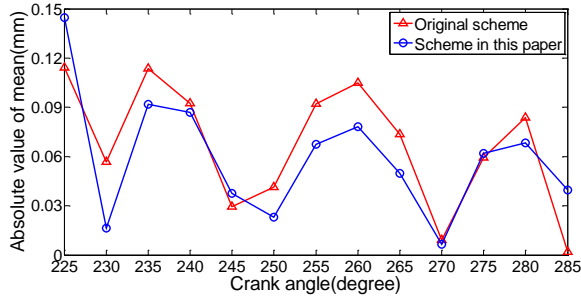


Fig. 6 The absolute value of the mean of error comparison

of original scheme and this paper scheme

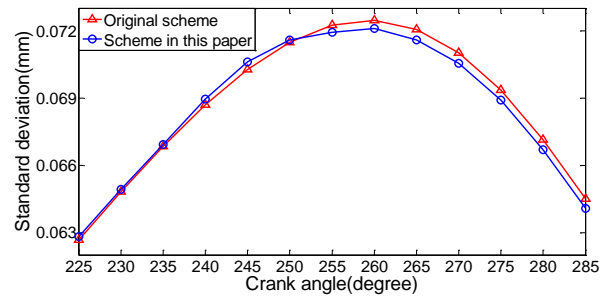


Fig. 7 The standard deviation of error comparison of

of original scheme and this paper scheme

As shown in **Fig.6** and **Fig.7**, compared with the result of the typical robustness design, the similar variation characteristics of the mean value and the standard deviation can be found, so the method in this paper is effective.

According to the design model of this paper, the design parameters and the motion error of three (minimum clearance111, medium clearance222 and maximum

clearance333) schemes were calculated, through the process in **Fig.3**, results of design parameters for three schemes are given in **Table 4**, and the mean value and standard deviation of the motion error are shown in **Fig. 8** and **Fig. 9**

Table 4 Comparison of three schemes optimization results

Design Variables	l_1 / mm	l_2 / mm	l_3 / mm	$\Delta l_1 / mm$	$\Delta l_2 / mm$	$\Delta l_3 / mm$
Scheme 111	57.798	164.089	70.345	0.068	0.057	0.044
Scheme 222	58.006	164.583	70.742	0.076	0.063	0.045
Scheme 333	57.826	164.071	70.261	0.069	0.051	0.081

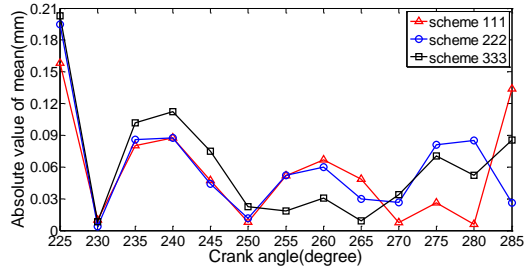


Fig. 8 The absolute value of mean of Error comparison

of three schemes

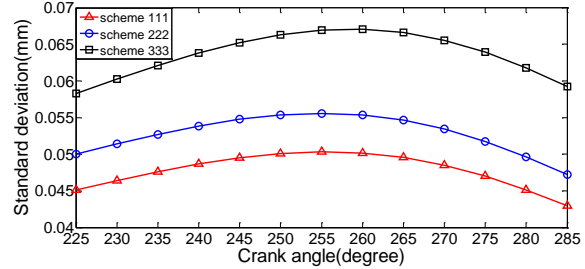


Fig. 9 The standard deviation of Error comparison of

three schemes

3 Multi-objective optimal design for motion precision and cost saving

In the engineering, the precision requirement is the precondition of the optimal design, and the cost is one of the main factors to be considered in the optimal design of planar linkage mechanism, but the motion precision of the mechanism and the cost of manufacturing cannot be effectively balanced.

In this paper, the optimal solutions are obtained

through the process of **Fig.3** is only the minimum value of the objective function. However, when the uncertainty factors increase, the whole precision of the mechanism will be affected [20-22], hence, it is necessary to evaluate the optimal solutions of the system, and a basis for the following cost requirements is provided.

3.1 Model precision and cost evaluation

In general design, the same fit clearance for each joint point is provided. However, in fact, some points have less

impact for the motion precision of the mechanism, if smaller clearances are applied to these points, the unnecessary cost will be increased, thus, in this paper, mainly aiming at this unnecessary cost to analyze.

Assuming each revolute pair of the slider-crank mechanism has three different clearances can be chosen (as shown in table1), that is, $3^3=27$ schemes in totally. Each scheme designed by the process in **Fig. 3**, and the 27 optimum solutions will be obtained, evaluating them according to the following precision evaluation criteria.

Supposing the object point is $o_i, i = 1, 2, \dots, 13$. Where the upper bound of the object expresses as $o_{ui} = o_i + o_i \cdot p$, the lower bound of the object expresses as $o_{li} = o_i - o_i \cdot p$. p is the object-precision (from **Fig.6** and **Fig.8** found that the mean value at the point of 225° is larger than the other point, hence, 0.2mm is used to defining the object-precision,

that is, $0.2/80=2.5\%$, namely the precision $p=2.5\%$). In the optimization results that we obtained, \bar{x}_i is the mean value of the motion error and σ_i is the standard deviation. Therefore, the upper bound of the optimization result is $x_{ui} = \bar{x}_i + \sigma_i, i = 1, 2, \dots, 13$. The lower bound is $x_{li} = \bar{x}_i - \sigma_i, i = 1, 2, \dots, 13$ [6-7]. Thus, the precision of the mechanism fulfills the requirements after optimization when $x_{ui} \leq o_{ui} \cup x_{li} \geq o_{li}, i = 1, 2, \dots, 13$.

When the optimization results of the schemes fulfill the precision requirements, then carry out cost evaluation, that is, the biggest sum of the sequence number are chosen (the bigger clearance causes the lower cost). Motion precision and cost evaluation process of the whole mechanism is shown in **Fig. 10**.

3.2 Multi-objective optimal design process and results

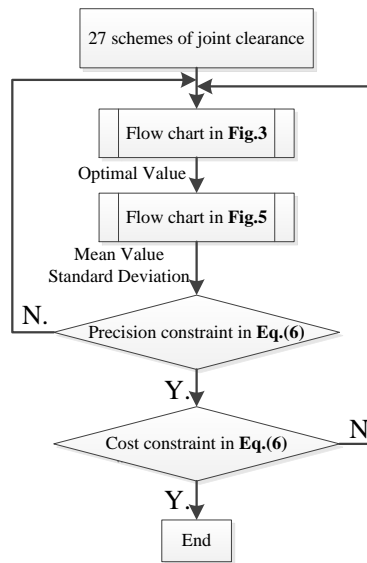


Fig. 10 Process analysis of multi-objective optimal design model

According to the process of multi-objective optimal design model in **Fig.10**, the 27 schemes are sampled calculation in turn, firstly, then the motion precision is evaluated and the schemes that fulfill the precision requirement are chosen, that is, 111,122,123,211,212,223,322 and 332. Finally, the biggest clearance scheme is chosen (lowest cost), that is, 332, thus,

the joint point of the actual mechanism is $\frac{H8}{e8}$ for point A,

$\frac{H8}{e8}$ for point B, and $\frac{H7}{f6}$ for point C, respectively.

In this way, the lowest manufacturing cost at guarantee the motion precision premise of this example can be obtained. Considering there are numerous factors affecting the cost, hence, the basic thought of balancing the precision and the cost is provided.

4 Conclusion

(1) In view of the uncertainty increases of the mechanism's whole motion error after manufacturing, predicting the motion error is difficult at the design phase. This paper analyses the relationship between design and

manufacturing, based on the Monte Carlo simulation method to presents an optimal design model of planar linkage mechanism with joint clearance.

(2) As shown in **Fig.6** and **Fig.7**, compared with the result of the typical robustness design, the similar variation characteristics of the mean value and the standard deviation can be found, so the method in this paper is effective.

(3) As shown in **Fig.8-Fig.9**, with the increase of the joint clearance, the mean value of the motion error fluctuates in a certain range. The standard deviation of the motion error increases, namely, the motion error fluctuation of the actual mechanism increases at the mean value points, the uncertainty of the mechanism's motion error increases. The model in this paper can be used to quantitatively describe the influence of different fit tolerance and dimensional tolerance on the motion error of the actual mechanism.

(4) In general, motion precision of the mechanism and the cost of manufacturing cannot be effectively balanced, that is, with the decrease of the joint clearance, the motion precision is increased, and the cost of the manufacturing is increased. In this paper, aiming at the typical offset slider-crank mechanism, three different clearances for each joint point is provided. Applying the model of this paper, 27 schemes are calculated, the results are analyzed, and a scheme meets the requirement of both the precision and the cost can be obtained. Considering there are numerous factors affecting the cost, hence, the basic thought of the balance of precision and the cost is provided.

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References:

[1] DONG yu-ge, NI zheng, ZHAO xian-de. A simple approach of the fuzzy reliability analysis of the slide-crank mechanism movement[J].Journal of Hefei University of technology,2004,27(11):1448—1452.(in Chinese)
[2] DONG yu-ge, CHEN Xinzhaoh, ZHAO xian-de. An application of fuzzy reliability theory in the reliability

analysis of the mechanism movement[J].Journal of Applied Science,2002,20(03):316—320.(in Chinese)
[3] LEE S J,GILMORE B J. The Determination of the probabilistic properties of velocities and acceleration in kinematic chains with uncertainty[J].Journal of Mechanical Design(1991),113(1):84—90.
[4] SONG li, YANG jian. Effect of dimension type on error transfer coefficient of planar linkage with clearance[J].Natural Science Journal of Xiangtan University,1999,21(3):64-69.(in Chinese)
[5] TAN xiao-lani, HAN jian-you, CHEN li-zhou. Robust Design of Function Generating Mechanisms Considering Kinematic Pair Clearance[J].Journal of University of Science and Technology Beijing,2004,26(4):416-419.(in Chinese)
[6] Kang Luo, Jing Wang, Xiaoping Du. Robust mechanism synthesis with truncated dimension variables and interval clearance variables [J].Mechanism and Machine Theory (2012) 71–83.
[7] Kang Luo, Xiaoping Du. Probabilistic mechanism analysis with bounded random dimension variables [J].Mechanism and Machine Theory (2013) 112-121.
[8] Xiaoping Du, Zhen Hu, First order reliability method with truncated random variables [J]. ASME Journal of Mechanical Design 134 (9) (2012) 091005-1–091005-9.
[9] Zhen Hu, Xiaoping Du. Time-dependent reliability analysis with joint upcrossing rates [J].Structural & Multidisciplinary Optimization, 2013, 48(5):893-907
[10] Gunesh R.Gogate, Sanjay B.Matekar. Optimum synthesis of motion generating four-bar mechanisms using alternate error functions [J].Mechanism and Machine Theory (2012) 41–61.
[11] Jianmin Zhu, Kwun-Lon Ting. Uncertainty analysis of planar and spatial robots with joint clearances [J].Mechanism and Machine Theory (2000) 1239-1256.
[12] Kwun-Lon, Jianmin Zhu. Ting The effects of joint clearance on position and orientation deviation of linkages and manipulators[J]. Mechanism and Machine Theory (2000) 391-401.
[13] GUO hui-xin, YUE wen-hui. Design Optimization of Planar Linkage Mechanism with Joint Clearance for Improving Robustness of Kinematic Accuracy [J]. Journal of Mechanical Engineering, 2012,03:75-81.(in Chinese)
[14] GUO hui-xin. Position robust analysis and design optimization of needle bar mechanism with joint

clearance of automatic embroidery machine [J]. Journal of Textile Research, 2011, 32: 131-136. (in Chinese)

[15] CHEN li-zhou, YU bi-qiang. Mechanical Optimal Design Method[M].Beijing: Metallurgical Industry Press,2014.(in Chinese)

[16] CHEN da-xian. Handbook of Mechanical Design[M].Beijing: Chemical Industry Press,2013.(in Chinese)

[17] LV cheng, LIU zi-jian, AI yan-di, YU zhi-min. Assembly Joint Surface Error Modeling and Tolerance Optimization in the Case of Coupled Tolerance[J].Journal of Mechanical Engineering, 2015,18:108-118.(in Chinese)

[18] MO xu-hui, ZHAO yu-hang, ZHONG zhi-hua, ZHANG yi. Robustness optimization of ride comfort for vehicle based on 6σ method [J]. Journal of Central South University(Science and Technology), 2012,11:4286-4292.(in Chinese)

[19] YUAN gui-xing, WANG ping. Monte Carlo Simulation and Its Application in Tolerance Design [J]. Journal of Tianjin University of Science&Technology,2008,02:60-64.(in Chinese)

[20] SUN zhi-li, JI guang-zhen, YAN yu-tao, YANG qiang. Mechanism motion reliability design and analysis technology [M]. Beijing: Nation Defense Industry Press,2015.(in Chinese)

[21] Sangmun Shin, Pauline Kongsuwon, Byung Rae Cho. Development of the parametric tolerance modeling and optimization schemes and cost-effective solutions [J]. European Journal of Operational Research (2010) 1728-1741.

[22] Xianzhen Huang, Yimin Zhang. Robust tolerance design for function generation mechanisms with joint clearances [J]. Mechanism and Machine Theory (2010) 1286-1297.

[23] H Zhao, X Xu. Model of flight technical error in symmetrical plane for performance based navigation [J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2011, 28(3):246-254.

[24] Yao Yu, Wu Hongtao. Force Jacobian Matrix for 3-DOF Cable-Driven Mechanism with Rotation in Wind Tunnel [J]. Journal of Nanjing University of Aeronautics&Astronautic, 2011, 43(1):75-78.

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